

# Photon Number Statistics of Multimode Parametric Down-Conversion

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We experimentally analyze the complete photon number statistics of parametric down-conversion and ascertain the influence of multimode effects. Our results clearly reveal a difference between single mode theoretical description and the measured distributions. Further investigations assure the applicability of loss-tolerant photon number reconstruction and prove strict photon number correlation between signal and idler modes.

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The photon statistics of a light beam are the “fingerprint” of its quantum state, from which a number of useful measures of non-classicality may be inferred. Coherent states reflect closest the classical properties of optical fields. These states exhibit a Poissonian photon number distribution, while sub-Poissonian statistics are a clear signature of quantum light. On the other hand, single-mode thermal radiation shows super-Poissonian photon statistics. However, thermal light sources typically emit a multimode field so that their photon statistics are given by the convolution of several thermal photon number distributions, which, in turn, leads again to a Poissonian distribution. In some applications, such as quantum key distribution, the photon statistics and their interplay with the multimode structure are intrinsically important for determining the security of the communication.

For multi-partite systems another important quantum characteristic is the extent to which they are correlated beyond what is possible classically. Quantum correlations, or entanglement, underlie much of the information processing power of quantum systems. For multiple light beams it is therefore important to determine the correlations between the different modes, and photon-detection methods that can reveal these correlations become paramount. The joint photon statistics of two beams can provide a useful measure of correlation and the degree of entanglement, also important for applications such as decoy state quantum cryptography [1].

In this Letter we report, to our knowledge, the first direct detection of complete higher order photon number correlations and statistics between two light beams that are strictly correlated in photon number (a twin-beam), using a photon-number-resolved characterization whose performance is compromised neither by dark-count noise nor by detector inefficiencies. This enables us to detect strict photon number correlations for all observed photon numbers including the vacuum. We are also able to identify features caused by the multimode character of the twin-beam source used.

In general, photodetectors are not intrinsically mode-selective, since they have a finite étendue and spectral response bandwidth. In this regard they are ideal for

measurements in the photon-number basis. However, detectors that are sensitive enough to respond to single photons are usually unable to resolve the differences between small numbers of incident photons. In contrast, detectors for large photon fluxes usually have high efficiency, but are too noisy to be sensitive to individual photons. Hence, there currently exist two main approaches to recover photon number statistics.

The first approach, homodyne detection, employs an ancillary optical beam, the local oscillator, to increase the photon flux level so that it may be registered on a standard photodiode. The photon statistics needs to be recovered indirectly from a large data set by means of quantum state tomography [2] or the use of pattern functions [3]. Therefore the photon number statistics are only recovered indirectly involving the acquisition of a large data set. Further, the local oscillator necessarily acts as a mode filter, picking out the component of the input beam that matches the mode of the ancilla. Hence, the multimode character becomes obscured by the detection technique. Although multimode homodyne detection is possible [4], it is laborious and requires a very high signal to noise ratio to accurately estimate the photon numbers.

The second approach combines mode-multiplexing with single-photon sensitivity to allow for multimode state characterization without the need of a local oscillator. However, achieving single-photon sensitivity, high quantum efficiency and low dark count noise constitutes an experimental challenge. The two most common types of detectors for such applications are avalanche photodiodes (APDs) and visible-light photons counters (VLPCs). VLPCs can detect single photons and achieve multi-photon resolution by spatial mode-multiplexing. They have been used to determine the non-Poissonian photon number statistics of degenerate parametric down-conversion (PDC), to analyze the statistics of heralded number states using twin beams, and to verify the non-classicality of the PDC states [5]. While VLPCs exhibit very high quantum efficiencies, they suffer from significant dark count contributions and require cryogenic cooling. Similar difficulties are encountered using sensitive CCD cameras at room temperature [6]. Photon number

resolved measurement can also be achieved combining the idea of mode-multiplexing with the ease of operation offered by commercial APDs. Here an input pulse is distributed in the time domain into multiple temporal modes using an optical fiber network [7]. In 2006 Achilles *et al.* introduced a new technique for loss-tolerant characterization of photon number statistics [8] using a time multiplexed detector (TMD) to characterize the conditioned statistics of one- and two-photon states from a twin beam.

Generation of twin beams by means of PDC has become a widespread way to implement highly correlated quantum states. Sub- and super-Poissonian photon number statistics [9], photon anti-bunching [10] and quantum correlated quadrature amplitudes [11] have been demonstrated. More recently, a PDC source was employed to prepare "kitten" states by means of photon subtraction [12], paving the way for a new approach to conditional quantum state synthesis.

Although some measurements of joint photon number statistics of PDC have been performed, a complete and accurate measurement of the joint photon number probability  $p_{n,m}$  for two beams had not been achieved. Such measurements should quantify the photon number correlations between the signal and idler beams to all orders, and determine the moments of the unconditional (marginal) and conditional photon distributions for each beam. Here, we show such a set of measurements. We employ two TMDs for detecting simultaneously signal and idler beams, for a range of parametric gains. We are able to build up the full joint, marginal and conditional photon number distributions, and to confirm various important correlations. The consistency of the reconstructed probability distributions across a variety of measurements validates our approach, and enables us to measure directly the photon number distribution even for highly multimode beams.

Most crucial for loss-tolerant reconstruction of the photon number distribution of a twin beam state  $|\Psi\rangle = \sum_n \sqrt{p_n} |n, n\rangle$  is a knowledge of losses, or equivalently the detection efficiency. Calibration according to the extended Klyshko method [7, 8] requires the a priori assumption of photon number correlation. This assumption, however, needs verification, and we are able to show this experimentally using the TMDs.

At the heart of our experiment is a TMD, whose response [13] for incoming photon statistics  $\vec{p} = (p_0, \dots, p_n, \dots)$  is modeled as

$$\vec{q} = \mathbf{C}\mathbf{L}(\eta)\vec{p} \quad (1)$$

$$\mathbf{L}(\eta) = L_{n,m} = \binom{m}{n} \eta^n (1-\eta)^{m-n}. \quad (2)$$

The mathematical description includes a loss matrix  $\mathbf{L}(\eta)$  and a convolution matrix  $\mathbf{C}$ . The loss relates the individual survival probability  $\eta$  of one photon to the survival

of  $n$  out of  $m \geq n$  photons. The convolution accounts for a probabilistic distribution of  $n$  photons into  $k$  different time-multiplexed modes whilst taking into account an individual population probability  $P_k$ . Both matrices act on a photon probability vector  $\vec{p}$  and yield an observable click vector  $\vec{q}$  in the detector. The probability matrices  $\mathbf{L}(\eta)$  and  $\mathbf{C}$  act non-deterministically on the measurement outcome on a shot-by-shot basis. Nevertheless, they provide an analytic and deterministic functional dependence for state reconstruction when applied to an ensemble measurement. For accurate photon number measurements, all model parameters  $\eta$ ,  $P_k$  and  $\vec{q}$  need to be experimentally accessible.

Our experimental setup is depicted in Figure 1. A diode laser source at 404nm pulsed with a repetition rate of 1MHz, with spectral and temporal bandwidths of 2.0nm and 60ps FWHM respectively, undergoes mode cleaning, followed by polarization control. It is subsequently coupled into a periodically poled KTiOPO<sub>4</sub> waveguide chip by means of a focussing graded index lens. The nonlinear interaction of pump photons decaying into signal and idler photons via a type-II PDC process (at central wavelengths  $\lambda_s = 740\text{nm}$ ,  $\lambda_i = 890\text{nm}$ ) results in the generation of orthogonally polarized, photon number correlated twin beams.

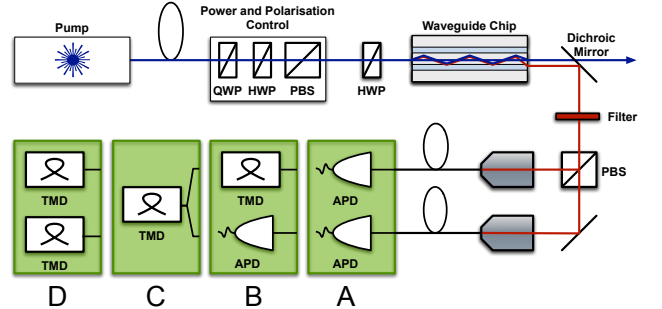


FIG. 1: Scheme of experimental setup (see text); measurement configurations A, B, C and D provide information about detector efficiency, marginal and collective statistics, and joint correlations respectively.

After state preparation, a dichroic mirror in combination with a red pass filter splits the PDC state from the strong pump. A polarizing beam splitter separates signal and idler modes, which are then coupled into multimode fibers for detection with different APD or TMD measurement configurations, labeled as A, B, C and D in Figure 1. These setups are fiber pigtailed for easy interchangeability with minimal influence on the detection efficiency. Tight temporal filtering with precision of 82ps is applied during data acquisition for noise reduction. The experiments are performed at cw-equivalent powers of pump light coupled through the waveguide, ranging from 40nW to 375nW.

The different measurement configurations are used for

a sequence of experiments that allow for checking the consistency of the recovered information about the state. We begin with determining the detection efficiency and then use this calibration to analyze the marginal statistics of signal and idler mode respectively. The measurement of the collective statistics of both modes is then followed by the complete measurement of the joint photon number statistics of the PDC source.

Setup A provides the basis for measuring the efficiency  $\eta_K = \frac{R_C}{R_S}$  from the count rates of coincidence ( $R_C$ ) and single ( $R_S$ ) events. We calibrate the real efficiency  $\eta$  by evaluating  $\eta_K$  at the lowest pump power of 40nW and observe signal and idler efficiencies of  $\eta_s = 11.7\%$  and  $\eta_i = 13.7\%$  respectively. Our PDC shows high source brightness even at this low pump power, yielding count rates of 4950/s and 5764/s for signal and idler.

In our second configuration, setup B, we study the marginal statistics of the signal and idler mode. We replace, for example, the idler-APD from the previous setup by a TMD in order to obtain the statistics of the idler mode. A knowledge of  $\eta_i$  is crucial for the loss tolerant reconstruction of photon statistics. We infer  $\eta_i$  from a coincidence measurement with the remaining APD in the signal mode. We introduce an additional fiber coupler to run this measurement and find a decline in efficiency to  $\eta_i = 11.3\%$ . We extract all parameters of our model directly from the measured data: the TMD click statistics  $\vec{q}$  on a shot-by-shot basis; the efficiency  $\eta$  from coincidences between idler-TMD and signal-APD for loss tolerant reconstruction via  $\mathbf{L}$ ; the occupation probability  $P_k$  of a particular TMD time-mode defining the convolution  $\mathbf{C}$ . The reconstructed photon statistics are obtained using a direct matrix inversion of Eq.1 [8]. The resulting marginals are shown in Figure 2a for a range of pump powers, displaying several photon statistics with respective values  $p_0, \dots, p_4$ , which correspond to the growth of mean photon numbers. Figures of merit reflecting the same information are easily computed, such as the probability moments  $\langle n^m \rangle$ , depicted in inset of Figure 2a and compared to a Poissonian fit. Reconstructed marginals for TMD measurements in signal and idler beams are presented in Figure 2b. The marginals show a good agreement for all photon numbers between signal and idler, which validates our photon number reconstruction as this is expected for photon number correlation. The conditioned statistics (Figure 2b inset) are needed for the calibration and confirm the preparation of heralded single photon states for the lowest pump power as well as the good suppression of vacuum contributions for all pump powers.

Using setup C, we extend our testing of the consistency of the inversion method. In conjunction with the previous measurements, we now compare signal marginals, idler marginals and the collective statistics of the twin beams. This experiment comprises the collective coupling of both signal and idler beams into a single TMD. Thus, the con-

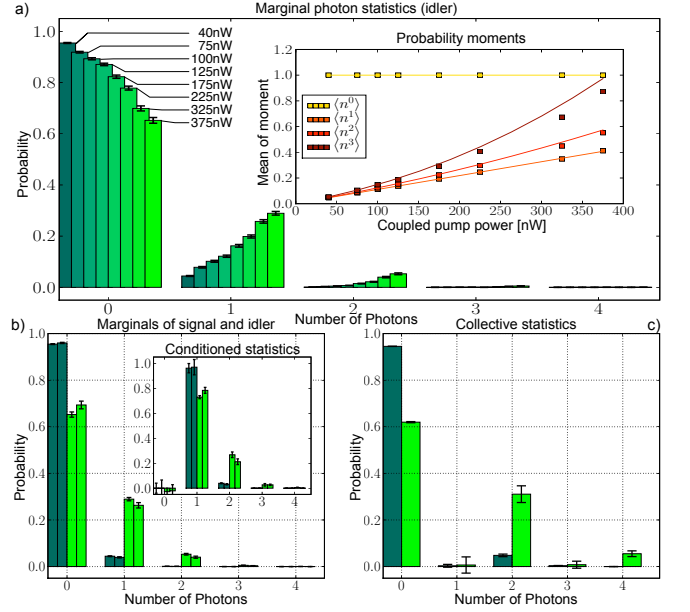


FIG. 2: (a) Marginal photon number statistics in idler mode for various pump powers; Inset: first probability moments  $\langle n^m \rangle$  with a fit to a Poissonian distribution. (b) Comparison of signal and idler marginals for lowest and highest pump power; Inset: conditioned statistics demonstrate vacuum suppression. (c) Collective photon pair statistics (see text) for lowest and highest pump powers.

figuration renders a strict test for the occurrence of photon pairs. The suppression of odd photon numbers, presented in Figure 2c, provides direct evidence for photon number correlations. Clearly, this configuration cannot be used to measure coincidences between signal and idler photons. We therefore used the calibration of  $\eta_s$  and  $\eta_i$  from previous measurement configurations. As expected, the photon statistics exhibit oscillations in the collective probability distribution, arising from the pairwise generation of photons. Further, comparing the collective statistics to the marginal distributions of Figure 2b we can prove that the distributions are consistent across all measurements, because  $p_{2n}^{\text{collective}} = p_n^{\text{signal}} = p_n^{\text{idler}}$  holds.

We complement our state characterization with setup D and demonstrate the first direct joint photon number measurement by simultaneously operating two TMDs, one for each beam. This enables us to measure directly all photon number correlations between signal and idler. The theoretical model of a single TMD can easily be extended to cover multiple TMDs. Let us assume that signal and idler modes obey a photon number distribution for which  $p_{n,m}$  is the probability to find  $n$  and  $m$  photons in signal and idler beams respectively. Each beam is analyzed by one TMD independently:

$$\vec{q}_{k,l} = (\mathbf{C}_s \mathbf{L}_s(\eta_s) \otimes \mathbf{C}_i \mathbf{L}_i(\eta_i)) \vec{p}_{n,m} \quad (3)$$

Experimentally we find that the probability correlation matrix  $p_{n,m}$  is dominantly populated on the diagonal,

which proves the strict photon number correlation between signal and idler. This, in turn, is a strong justification for exploiting the a priori assumption of pair generation for our source, used for calibrating the detector efficiency. In this experiment we coupled the signal beam into a single mode (SM)-fiber TMD, and the idler beam into multi mode (MM)-fiber TMD. The electronics records correlated click statistics between both TMDs. The coupling into SM-fibers comes along with a decrease of detection efficiency to a level of  $\eta_s = 2.74\%$  whereas  $\eta_i = 11.1\%$  remains unchanged. Our setup is sufficiently loss tolerant, though, to cope with the low efficiency. The result  $p_{n,m}$  for a measurement with pump power of 220nW is depicted in Figure 3. Slight uncertainties in the measured click statistics due to finite acquisition time of 300s and the modest efficiency in the signal mode contribute to minute errors found in off-diagonal elements. From this data we determine a normalized covariance of 0.996 in the photon number statistics, compared to 0.052 in the raw data. The reconstructed twin beam photon number squeezing  $\frac{\Delta^2(n_s - n_i)}{\langle n_s \rangle \langle n_i \rangle}$  is found to be  $-23.9\text{dB}$  for the photon number statistics and  $-0.2\text{dB}$  for the raw data.

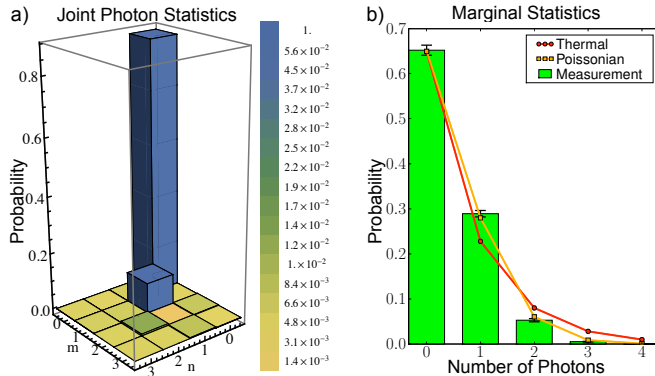


FIG. 3: (a) Joint photon statistics  $p_{n,m}$  measured with two TMDs and pump power of 220nW. (b) Marginal statistics of idler with pump power of 375nW. The measurement is fitted against thermal and Poissonian distributions.

At high power levels we find a mismatch between the observed photon statistics and the predictions from a single-mode theory of the PDC process, equally described as a two-mode squeezer. The evaluation of the quadratic Hamiltonian gives for the signal and idler mode a thermal marginal distribution  $p_n = \frac{\langle n \rangle^n}{(1 + \langle n \rangle)^{n+1}}$ . In contrast, our results obey a Poissonian marginal distribution  $p_n = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!}$ , as depicted in Figure 3, and illustrated by the fits. The thermal fit misses the observation in the one-photon contribution by 0.0614, which is more than seven times larger than the statistical error. This error is derived analytically by Gaussian error propagation applied to Eq. 1 taking into account uncertainties in each model parameter. The measurement uncertain-

ties for the efficiencies are 0.009, and for the click statistics a conventional square root dependence is assumed. The discrepancy between the observed Poissonian statistics and the thermal distribution of a two-mode squeezed state can only be explained by a PDC process, introducing a multitude of different modes. The convolution of many independent thermal statistics yields the observed Poissonian statistics.

In summary, we have demonstrated the first explicit measurement of correlations and complete joint photon number statistics of a PDC source. Our approach to use a TMD for loss-tolerant state characterization enabled us to study the marginal photon number distributions in detail. This revealed a clear signature of a multimode structure of the source. We expect the multimode structure of light to play an increasing role for the development of advanced quantum key distribution protocols as it could be exploited for implementing higher dimensional information coding [14]. Multimode states need to be considered for developing hybrid quantum communication schemes that include continuous variable states as well as single photons [15].

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